NAVAL WAR COLLEGE
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[redacted]

"PROBABILITY CONCEPTS FOR GAMING"

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I. <u>INTRODUCTION</u>.

GOOD MORNING, GENTLEMEN

SLIDE 1 ON

THIS GAMBLER, WHO IS PLAYING POKER, IS INVOLVED IN A CONFLICT SITUATION IN WHICH HIS OPPONENTS ARE READY AND WILL TO EXPLOIT HIS MISTAKES AND ADVERSITIES. EVENTS OCCUR IN THIS GAME ACCORDING TO A SET OF RULES, WHICH ARE INTRICATELY WOVEN ABOUT CERTAIN NATURAL LAWS CALLED THE "LAWS OF PROBABILITY." THE GAMBLER DOES NOT HAVE TO KNOW THESE "LAWS OF PROBABILITY" TO PLAY THE GAME, BUT HE CERTAINLY WILL DO MUCH BETTER IF HE DOES KNOW THEM. THIS IS ESPECIALLY TRUE IF HIS OPPONENTS ARE FAMILIAR WITH THE LAWS, AND HE IS NOT.

SLIDE 1 OFF

WAR IS ALSO A CONFLICT SITUATION IN WHICH OPPONENTS STRIVE TO EXPLOIT EACH OTHERS SHORTCOMINGS. COMBAT OPERATIONS ARE EVENTS THAT DETERMINE THE COURSE OF THE WAR, AND THEY ARE ALSO WOVEN ABOUT THE "LAWS OF PROBABILITY." THE RELATIONSHIPS BETWEEN EVENTS IN COMBAT OPERATIONS AND THE "LAWS OF PROBABILITY" ARE NOT AS CLEARLY DEFINED AS THEY ARE IN THE POKER GAME, BUT THEY ARE NO LESS IMPORTANT. FOR PLANNING PURPOSES, IT WOULD BEHOOVE US TO UNDERSTAND THESE RELATIONSHIPS IN ORDER TO ANALYSE WHAT HAS HAPPENED, AND WHAT MAY HAPPEN IN COMBAT OPERATIONS. BUT WHAT IS THE CONNECTION WITH WAR GAMING? WELL, WAR GAMING SIMULATES COMBAT OPERATIONS. IF EVENTS OCCUR IN A

PROBABILISTIC FASHION IN REAL COMBAT OPERATIONS THEN THEY MUST OCCUR IN AN SIMILAR MANNER IN WAR GAMING. TO ESTABLISH AN ENVIRONMENT IN WAR GAMING WHICH SIMULATES THE PROBABILISTIC OCCURRENCE OF EVENTS, CERTAIN CONCEPTS, GOVERNED BY THE LAWS OF PROBABILITY, ARE EMPLOYED.

THESE CONCEPTS ARE REALLY QUITE SIMPLE, AND DEPEND ONLY UPON SOME FUNDAMENTAL PROBABILITY LAWS.

LIKE THE GAMBLER, WE WILL BETTER UNDERSTAND A WAR GAME IF WE ARE FAMILIAR WITH THESE CONCEPTS, AND THE MANNER IN WHICH THEY AFFECT THE OUTCOME OF EVENTS.

SLIDE 2 ON

IT IS THE PURPOSE OF THIS LECTURE TO DEFINE "PROBABILITY", TO FAMILIARIZE YOU WITH THE TERMS USED IN CONNECTION WITH PROBABILITY, TO PROVIDE YOU WITH THE RULES FOR COMBINING PROBABILITIES IN MULTIPLE EVENTS, AND TO SHOW YOU SOME OF THE APPLICATIONS AND CHANCE DEVICES USED IN WAR GAMING.

SLIDE 2 OFF

LET US START BY DEFINING THE TERM PROBABILITY.

II. DEFINITIONS.

THERE ARE TWO WIDELY ACCEPTED DEFINITIONS OF PROBABILITY.

SLIDE 3 ON

THE FIRST ONE IS DEDUCED PRIOR TO ANY EVENT OCCURING AND IS CALLED THE "A PRIORI", OR "DETERMINED IN ADVANCE", PROBABILITY.

LET THE SYMBOL "E" REPRESENT AN EVENT. SUPPOSE THIS EVENT CAN OCCUR IN m WAYS OUT n EQUALLY LIKELY WAYS, AND CAN FAIL TO OCCUR IN $(n - m)$ WAYS OUT OF THE n WAYS. THEN WE SAY THAT THE PROBABILITY THAT THE EVENT WILL OCCUR IS m/n, AND THAT THE PROBABILITY THAT THE EVENT WILL FAIL TO OCCUR IS $\frac{n-m}{n}$. THE PROBABILITY THAT THE EVENT WILL FAIL TO OCCUR IS REFERRED TO AS THE "COMPLEMENTARY PROBABILITY", AND IS EQUAL TO 1 MINUS THE PROBABILITY THAT THE EVENT WILL OCCUR.

SLIDE 3 OFF

A SIMPLE EXAMPLE WILL EXPLAIN THIS DEFINITION.

SLIDE 4 ON

AN UNBIASED COIN IS TO BE TOSSED; AND WE WISH TO KNOW WHAT IS THE PROBABILITY THAT IT WILL FALL "HEADS". THE TOSS CAN RESULT IN TWO WAYS, "HEADS" OR TAILS". THE EVENT "E", THAT THE COIN WILL FALL "HEADS", CAN OCCUR IN ONE WAY. FAILURE OF EVENT "E", THAT THE COIN WILL FALL "TAILS", CAN OCCUR IN ONE WAY. THEREFORE, THE PROBABILITY THAT THE COIN FALLS "HEADS" IS m/n, OR ONE-HALF. THE PROBABILITY THAT THE COIN WILL FALL TAILS IS $\frac{n-m}{n}$, OR ONE-HALF, AS WELL. AND, THE COMPLEMENTARY PROBABILITY IS 1 MINUS THE PROBABILITY THAT THE EVENT OCCURS; AGAIN ONE-HALF.

SLIDE 4 OFF

THE SECOND DEFINITION OF PROBABILITY INVOLVES THE OBSERVANCE OF REPETITIVE TRIALS OF AN EVENT, AND IS CALLED THE "A POSTERIORI", OR "DETERM NED AFTERWARDS", PROBABILITY.

SLIDE 5 ON

SUPPOSE THAT n TRIALS ARE ACCOMPLISHED IN WHICH m OUT OF THE n TRIALS RESULTED IN THE OCCURRENCE OF EVENT "E", AND (n - m) OUT OF THE n TRIALS RESULTED NON-OCCURRENCE OF EVENT "E". THEN BASED ON THIS SERIES OF n TRIALS, THE <u>APPROXIMATE</u> PROBABILITY OF OCCURRENCE OF EVENT "E" OCCURS IS m/n.

NOW, AS MORE EXTENSIVE TRIALS ARE ACCOMPLISHED, THE APPROXIMATE PROBABILITY OF OCCURRENCE WILL CONVERGE ON THE THEORETICAL VALUE. THAT IS, IF n BECOMES INFINITE, THE APPROXIMATE PROBABILITY OF OCCURRENCE OF THE EVENT SHOULD CONVERGE TOWARD THE THEORETICAL, OR "A PRIORI", VALUE. IN ADDITION, THE COMPLEMENTARY PROBABILITY RELATIONS CAN BE DERIVED BY THE SAME REASONING, AND ARE SHOWN NEAR THE BOTTOM OF THE SLIDE.

FOR AN EXAMPLE OF THIS DEFINITION, LET US AGAIN LOOK AT THE COIN TOSSING EVENT. (PAUSE)

SLIDE 5 OFF, SLIDE 6 ON

IN THIS CASE, AN UNBIASED COIN IS TOSSED 500 TIMES. THE SERIES OF TRIALS IS REPEATED FOUR TIMES, AND THE NUMBER OF "HEADS" OBTAINED ARE RECORDED AS SHOWN ON THE SLIDE. ONE WOULD EXPECT THAT THE NUMBER OF "HEADS" OBSERVED WOULD EQUAL TO 250 IN EACH SERIES. OBSERVE THAT SUCH IS NOT THE CASE. THEN BASED ON THESE FOUR SERIES, <u>WHAT IS (?)</u> THE APPROXIMATE PROBABILITY THAT "HEADS" OCCURS? TAKING EACH SERIES PROGRESSIVELY, WE CAN COMPUTE THE APPROXIMATE PROBABILITY THAT "HEADS" OCCURS. THE BEST ESTIMATE OF THE THEORECTICAL PROBABILITY OF "HEADS" IS THE

APPROXIMATE PROABILITY FOR ALL OF THE DATA AVAILABLE, IN THIS CASE, THE FOUR SERIES. (PAUSE) NOTE THAT THE APPROXIMATE, OR EXPERIMENTAL PROBABILITY OF "HEADS" IS ALREADY CONVERGING TOWARD THE "A PRIORI" VALUE OF $\frac{1}{2}$. WITH ADDITIONAL SERIES OF TRIALS, IT WOULD BE EXPECTED THAT THE EXPERIMENTAL VALUE WOULD CONVERGE TOWARD THE THEORETICAL VALUE EVEN MORE CLOSELY. (PAUSE)

SLIDE 6 OFF

AT THIS POINT, YOU ARE PROBABLY WONDERING WHICH OF THE TWO DEFINITIONS TAKES PRECEDENCE. THE SITUATION IS ANALOGOUS TO THE QUESTION: "WHICH CAME FIRST - THE CHICKEN, OR THE EGG?" THE ANSWER IS THAT BOTH DEFINITIONS ARE CORRECT, AND THAT ONE SHOULD USE THE DEFINITION WHICH PROVIDES THE INFORMATION THAT HE REQUIRES. FOR EXAMPLE, IN MANY INSTANCES THE EXPERIMENTAL DEFINITION MUST BE USED BECAUSE THE THEORETICAL DEFINITION CANNOT BE DERIVED FROM THE KNOWLEDGE WHICH WE HAVE REGARDING THE EVENT. USUALLY, BOTH PROBABILITIES ARE VERY CLOSE TO EACH OTHER, AND FOR PURPOSES OF GAMING, CAN BE USED INTERCHANGEABLY.

SO FAR, PROBABILITIES OF A SINGLE EVENT HAVE BEEN DISCUSSED. LET US NOW LOOK AT THE MANNER IN WHICH PROBABILITIES COMBINE FOR MULTIPLE EVENTS. FOR PURPOSES OF EXPLANATION, THE ANALYSIS WILL BE RESTRICTED TO TWO EVENTS, ALTHOUGH THE CONCEPTS ARE APPLICABLE TO MORE THAN TWO EVENTS, AS I WILL INDICATE LATER ON.

III. COMBINATIONS OF PROBABILITIES FOR MULTIPLE EVENTS.

SLIDE 7 ON

RELATED EVENTS CAN BE CLASSIFIED AS TO HOW THEY CAN OCCUR. THEY CAN BE:

1. SIMULTANEOUS EVENTS: THAT IS, THEY CAN OCCUR TOGETHER.
2. MUTUALLY EXCLUSIVE EVENTS: WHICH MEANS THAT THE EVENTS CANNOT OCCUR TOGETHER, OR
3. STATISTICALLY INDEPENDENT EVENTS: WHICH MEANS THAT THE PROBABILITY OF OCCURRENCE OF ONE EVENT IS INDEPENDENT OF WHETHER OR NOT THE OTHER EVENT HAS OCCURRED.

THE LAST TWO CATEGORIES CAN BE CONSIDERED AS SPECIAL CASES OF THE FIRST CATEGORY.

SLIDE 7 OFF

WHENEVER TWO OR MORE EVENTS ARE CONSIDERED AT THE SAME TIME, IT IS NECESSARY TO ANALYSE THE DIFFERENT OUTCOMES THAT ARE POSSIBLE. CONSIDER, IF YOU WILL, TWO EVENTS "E" AND "F".

SLIDE 8 ON

USING THE NOTATION OF THE LETTER TO MEAN THAT AN EVENT OCCURS, AND THE LETTER WITH A BAR OVER IT TO MEAN THAT THE EVENT DOES NOT OCCUR, WE CAN LIST THE POSSIBLE OUTCOMES OF THE EVENTS WHEN CONSIDERED TOGETHER.

EF MEANS THAT BOTH E AND F OCCUR;

$E\bar{F}$ MEANS THAT E OCCURS, BUT F DOES NOT;

$\bar{E}F$ MEANS THAT F OCCURS, BUT E DOES NOT;

AND $\bar{E}\bar{F}$ MEANS THAT NEITHER E NOR F OCCUR.

SLIDE 8 OFF, SLIDE 9 ON

THESE OUTCOMES MAY BE PUT INTO TABULAR FORM WHICH MIGHT BE EASIER TO VISUALIZE. IN THE TABLE OF OUTCOMES, F AND $\bar{F}$ ARE TABULATED ACROSS THE TOP, AND E AND $\bar{E}$ ARE TABULATED DOWN THE LEFT-HAND SIDE (POINT). OCCURRENCE OF E AND F OCCURS IN THIS CELL (POINT (EF)), AND OCCURRENCE OF E BUT NON-OCCURENCE OF F OCCURS IN THIS CELL (POINT ($E\bar{F}$)). THIS SYMBOL (POINT (E)) REPRESENTS THE ABSOLUTE OUTCOME OF E, REGARDLESS OF WHETHER OR NOT F OCCURS. THE SYMBOL THAT GOES IN THIS CELL (POINT - ALL POSSIBLE OUTCOMES) REPRESENTS ALL POSSIBLE OUTCOMES.

THE TABLE OF OUTCOMES PROVIDES THE SYMBOLS REQUIRED TO DEFINE THE FUNDAMENTAL PROBABILITIES ASSOCIATED WITH MULTIPLE EVENTS.

FIRST OF ALL, THERE IS:

P(E): THE ABSOLUTE PROBABILITY THAT E OCCURS.

P(F): THE ABSOLUTE PROBABILITY THAT F OCCURS.

NEXT, THERE IS:

P(E/F): THE CONDITIONAL PROBABILITY THAT E OCCURS, PROVIDED THAT F HAS OCCURRED.

P(F/E): THE CONDITIONAL PROBABILITY THAT F OCCURS, PROVIDED THAT E HAS OCCURRED.

THERE IS:

P(EF): THE COMPOUND PROBABILITY THAT BOTH E AND F OCCUR.

LASTLY, THERE IS:

P(E+F): THE PROBABILITY THAT E OR FO(<u>OR BOTH</u>) OCCUR.

IT IS POSSIBLE TO FIND MATHEMATICAL RATIOS FOR EACH OF THESE PROBABILITIES IN TERMS OF THE FREQUENCY OF OCCURRENCE OF THE DIFFERENT OUTCOMES. HOWEVER, IT IS NOT TO OUR INTERESTS TO DO SO THIS MORNING.

BY CERTIN ALGEGRAIC MANIPULATIONS OF THE MATHEMATICAL EQUIVALENTS OF THESE FUNDAMENTAL PROBABILITIES, DERIVED RELATIONSHIPS RESULT.

SLIDE 9 OFF

THESE, DERIVED RELATIONSHIPS EXPRESS THE VARIOUS PROBABILITIES IN TERMS OF EACH OTHER AND ARE IMPORT IN THE PRACTICAL USAGES OF PROBABILITIES.

SLIDE 10 ON

TAPE 005

FOR THE CASE OF SIMUL~~ATION~~ EVENTS, EVENTS WHICH CAN OCCUR TOGETHER, THE PROBABILITY THAT E AND F OCCUR IS EQUAL TO THE CONDITIONAL PROBABILITY THAT ONE OF THE EVENTS OCCURS, GIVEN THAT THE OTHER EVENT HAS OCCURRED, TIMES THE ABSOLUTE PROBABILITY THAT THE OTHER EVENT OCCURS (POINT - P(E/F) P(F)).

THE PROBABILITY THAT E <u>OR</u> F OCCURS IS EQUAL TO THE SUM OF THE ABSOLUTE PROBABILITIES OF THE EVENTS OCCURRING (POINT-P(E)+P(F)), MINUS THE PROBABILITY THAT THEY OCCUR TOGETHER (POINT-P(EF)).

WHEN THE EVENTS ARE MUTUALLY EXCLUSIVE, THEY CANNOT OCCUR TOGETHER AND THE PROBABILITY THAT THEY DO, IS ZERO (POINT - P(EF)). THE PROBABILITY THAT E OR F OCCURS, IN THIS SITUATION,

IS THE SAME AS BEFORE EXCEPT THE TERM FOR THE COMPOUND EVENT IS ZERO (POINT - P(E + F)). ALSO, THE CONDITIONAL PROBABILITIES ARE ZERO, AS ONLY ONE OF THE EVENTS CAN OCCUR (POINT - P(E/F), P(F/E)).

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WHEN THE EVENTS ARE STATISTICALLY INDEPENDENT, THAT IS, THE OCCURRENCE OF ONE EVENT IS INDEPENDENT OF THE OTHER EVENT, THE PROBABILITY THAT THEY OCCUR TOGETHER IS THE PRODUCT OF THE ABSOLUTE PROBABILITIES THAT THEY OCCUR SEPARATELY (POINT - P(E) P(F)). THIS TERM CAN BE SUBSTITUTED FOR THE COMPOUND EVENT TERM IN THE RELATIONSHIP FOR DETERMINING THE PROBABILITY THAT EITHER E OR F OCCURS (POINT - P(E + F)). THE CONDITIONAL PROBABILITY THAT ONE EVENT OCCURS GIVEN THAT THE OTHER HAS OCCURRED, IS SIMPLY THE ABSOLUTE PROBABILITY THAT THE FIRST EVENT OCCURS (POINT - P(E/F)). THIS IS BECAUSE THE EVENTS ARE INDEPENDENT, AND ONE EVENT HAS NO EFFECT ON THE OUTCOME OF THE OTHER.

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SLIDE 10 OFF

LET US LOOK AT SOME APPLICATIONS OF THESE RELATIONSHIPS. FOR AN EXAMPLE OF MUTUALLY EXCLUSIVE EVENTS, CONSIDER THIS MINING EXAMPLE.

SLIDE 11 ON

A SMALL PATROL SHIP IS PROCEEDING IN A CHANNEL IN WHICH THERE ARE TWO CONTACT MINES. THEY ARE DESIGNATED MINES E AND F TO CONFORM TO OUR PREVIOUS NOTATION. TWO ASSUMPTIONS WILL

BE MADE REGARDING THIS EXAMPLE, NAMELY, THAT THE SHIP WILL BE SUNK IF IT HITS A MINE, AND THAT THE MINES ARE SPACED SUCH THAT THE SHIP CANNOT HIT BOTH MINES DIMULTANEOUSLY.

NOW, - THE RECORD OF PREVIOUS SINKINGS OF THIS TYPE OF PATROL SHIP IN THIS CHANNEL REVEALS THAT THE APPROXIMATE PROBABILITY THAT THE SHIP WILL HIT A MINE IS ONE-SIXTH.

IF WE LET EVENT E REPRESENT THE SHIP HITTING MINE E, AND EVENT F REPRESENT THE SHIP HITTING MINE F, WE CAN CALCULATE THE PROBABILITIES, FOR MUTUALLY EXCLUSIVE EVENTS WHEN CONSIDERED TOGETHER.

HERE, WE USE THE FORMULAE FOR MUTUALLY EXCLUSIVE EVENTS. THE ABSOLUTE PROBABILITY THAT THE SHIP HITS MINE E IS ONE-SIXTH, AND THAT IT HITS MINE F IS ONE-SIXTH, ALSO (POINT). THE CONDITIONAL PROBABILITY THAT THE SHIP HITS A MINE HAVING ALREADY HIT THE OTHER IS ZERO, AS WE WOULD SURMISE (POINT - P(E/F) AND P(F/E)). THE COMPOUND PROBABILITY THAT THE SHIP HITS BOTH MINES IS ZERO, SINCE WE ASSUMED THAT THE SHIP COULD NOT HIT BOTH MINES SIMULTANEOUSLY (POINT - P(EF)). THE PROBABILITY THAT THE SHIP HITS EITHER MINE E OR MINE F IS THE SUM OF THE ABSOLUTE PROBABILITIES, ONE-THIRD (POINT), OR .33 IN DECIMAL.

SLIDE 11 OFF

THIS EXAMPLE OF THE PATROL SHIP IN THE MINED CHANNEL CAN BE USED FOR AN EXAMPLE OF STATISTICALLY INDEPENDENT EVENTS,

BUT WE MUST CHANGE THE ASSUMPTIONS. IF WE ASSUME THAT THE SHIP WILL BE DAMAGED INSTEAD OF BEING SUNK WHEN IT HITS A MINE, A DIFFERENT SITUATION IS GENERATED. NOW, ITS POSSIBLE FOR THE SHIP TO HIT BOTH MINES. THE PROBABILITY THAT THE SHIP WILL HIT ONE OF THE MINES IS INDEPENDENT OF THE FACT THAT IT HAS HIT THE OTHER MINE. THIS IS THE CRITERION FOR THE EVENTS TO BE STATISTICALLY INDEPENDENT.

THEREFORE, THE SITUATION HAS BECOME ON OF TWO EVENTS WHICH ARE STATISTICALLY INDEPENDENT. LET US LOOK AT THE PROBABILITY CALCULATIONS FOR THIS CASE WHERE WE USE THE FORMULAE FOR STATISTICALLY INDEPENDENT EVENTS.

SLIDE 12 ON

THE ABSOLUTE PROBABILITIES THAT THE SHIP HITS A SPECIFIED MINE IS ONE-SIXTH IN EACH CASE (POINT P(E) AND P(F)). THE CONDITIONAL PROBABILITIES THAT THE SHIP HITS ONE MINE, GIVEN THAT IT HAS HIT THE OTHER, IS EQUAL TO THE ABSOLUTE PROBABILITY (POINT P(E/F) AND P(F/E)). THE PROBABILITY THAT THE SHIP WILL HIT BOTH MINES, FOR THIS CASE, IS THE PRODUCT OF THE ABSOLUTE PROBABILITIES, OR ONE-THIRTY SIXTH (POINT P(EF)). THE LAST EXPRESSION IS THE PROBABILITY THAT THE SHIP HITS MINE E OR F, OR BOTH, AND IS POINT THREE ONE. THIS FIGURE WAS POINT THREE THREE FOR THE PREVIOUS MUTUALLY EXCLUSIVE EVENTS.

IT IS IMPORTANT TO NOTE THAT CERTAIN ASSUMPTIONS WERE MADE FOR THIS EXAMPLE, AND THAT A MINOR CHANGE TO ONE ASSUMPTION GENERATED A DIFFERENT SITUATION.

SLIDE 12 OFF

IN WAR GAMING IT IS OFTEN NECESSARY TO MAKE ASSUMPTIONS ABOUT SITUATIONS SIMILAR TO THIS EXAMPLE. FOR THIS REASON, WE MUST DESIGN THE GAME MODEL TO PORTRAY ACCURATELY THE DESIRED PROBABILISTIC ENVIRONMENT.

IN SOME WAR GAMES, THE OVERALL OUTCOME FOR AN EVENT IS A RESULT OF DETERMINING THE OCCURRENCE, OR NON-OCCURRENCE, OF THE COMPONENT ACTIONS AS THE EVENT PROGRESSES. THESE EVENTS ARE "PROBABILISTIC" EVENTS. IN OTHER GAMES, OUTCOMES FOR EVENTS HAVE ALREADY BEEN DETERMINED IN ADVANCE AND IN ACCORDANCE WITH THE RULES OF PROBABILITY. THESE EVENTS ARE "DETERMINISTIC" EVENTS. NOTICE THAT SEMANTICS PLAY A LARGE ROLE IN DISCUSSING PROBABILITIES. WHEN DISCUSSING PROBABILITIES, IT IS NECESSARY TO SPECIFY VERY PRECISELY WHICH PROBABILITY IS BEING CONSIDERED.

THUS FAR WE HAVE DISCUSSED ONLY TWO EVENTS. THE METHODS USED FOR TWO EVENTS CAN BE EXTENDED TO MORE THAT TWO EVENTS. SUPPOSE THAT THERE ARE FOUR EVENTS, E, F, G, AND H.

SLIDE 13 ON

THE COMPOUND PROBABILITY THAT ALL EVENTS WILL OCCUR IS THE PRODUCT OF THE CONDITIONAL PROBABILITIES, PROGRESSIVELY, AND THE ABSOLUTE PROBABILITY THAT THE FIRST EVENT WILL OCCUR. THIS IS AN EXTENSION OF THE RELATIONSHIP FOR TWO EVENTS.

IF THE EVENTS ARE ALL STATISTICALLY INDEPENDENT, THAT IS, THE PROBABILITY THAT ONE EVENT OCCURS IS INDEPENDENT OF THE OCCURRENCE OR NON-OCCURRENCE OF THE THE OTHER EVENTS, THEN THE COMPOUND PROBABILITY THAT ALL EVENTS WILL OCCUR IS THE PRODUCT OF THE ABSOLUTE PROBABILITIES THAT EACH EVENT OCCURS.

SLIDE 13 OFF; SLIDE 14 ON.

HERE IS AN EXAMPLE OF THIS NOTION. IT IS A CAP MISSION CONSISTING OF SEQUENTIAL EVENTS.

LET:

EVENT E REPRESENT THE LAUNCH OF CAP FROM A CARRIER,

EVENT F REPRESENT THE TALLY-HO OF THE CAP WITH A RAID,

EVENT G REPRESENT THE CAP PERFORMING A FIRING RUN,

AND EVENT H REPRESENT THE CAP KILLING THE RAID WITH A MISSILE.

FURTHERMORE, SUPPOSE THAT THE VALUES FOR THE ABSOLUTE PROBABILITY AND CONDITIONAL PROBABILITIES ARE AS INDICATED. THEN, THE COMPOUND PROBABILITY THAT ALL OF THE EVENTS OCCUR IS THE PRODUCT OF ALL OF THESE VALUES AND IS .57. THEREFORE, WITH THESE ASSUMPTIONS, THE PROBABILITY OF A SUCCESSFUL CAP MISSION IS .57.

IT IS IMPORTANT TO NOTE THAT THE COMPOUND PROBABILITY IS A RESULT OF MULTIPLYING THE CONDITIONAL PROBABILITIES AND THE ABSOLUTE PROBABILITY. MORE IMPORTANT, PERHAPS, IS THE OBSERVATION THAT THE HIGH INDIVIDUAL PROBABILITIES, WHEN COMPOUNDED, RESULT IN A RELATIVELY LOW OVERALL PROBABILITY OF SUCCESS. YOU PROBABLY WILL OBSERVE THIS EFFECT REPEATEDLY DURING THE

COURSE OF YOUR WAR GAMES HERE. RELIABILITY OF COMPONENT WEAPONS SYSTEMS, IF NOT 100%, CAN FURTHER REDUCE THE OVERALL PROBABILITY OF SUCCESS.

AT THIS POINT, LETS LOOK MORE CLOSELY AT A COUPLE OF ITEMS IN THIS EXAMPLE. FIRST OF ALL, NOTICE THAT THE EVENTS OF CAP LAUNCH AND MISSILE KILL ARE FAIRLY MUCH A FUNCTION OF EQUIPMENT PERFORMANCE. THE "HUMAN TOUCH" DOES NOT DIRECTLY AFFECT THESE EVENTS TO ANY GREAT DEGREE. HOWEVER, THE EVENTS OF SUCCESSFUL TALL-HO, AND PERFORMING A FIRING RUN, REFLECT THE PROFICIENCIES OF CONTROLLERS AND PILOTS. THAT IS, THEY REFLECT THE STATE OF TRAINING OF CONTROLLERS AND PILOTS. THEREFORE, BY ARBITRARILY ASSIGNING DIFFERENT PROBABILISTIC VALUES TO THESE EVENTS IN WAR GAMES, WE COULD INJECT THE STATE OF TRAINING FOR DIFFERENT UNITS INTO OUR GAMES. THIS PRINCIPLE MAY BE UTILISED IN OTHER SITUATIONS, AS WELL. NORMALLY, AN AVERAGE VALUE IS USED IN ANY GIVEN WAR GAME HERE AT THE WAR COLLEGE.

THIS BRINGS US TO THE SECOND ITEM - THAT OF INPUT VALUES USED. THE FIGURE OF .57 FOR THE PROBABILITY OF A SUCCESSFUL MISSION IS PURELY HYPOTHETICAL. OPERATIONS EVALUATION GROUP ANALYSES OF ACTUAL FLEET EXERCISE DATA, AND AIR-TO-AIR MISSILE TEST FIRINGS, INDICATE THAT THIS VALUE IS MUCH LOWER IN REALITY. THESE ANALYSES COVER A WIDE SPECTRUM OF EXERCISES AND PROFICIENCIES, AND INCLUDE THE "GOOD" AND THE "BAD".

SLIDE 14 OFF, SLIDE 15 ON

THE SITUATION IS SIMILAR TO THESE TWO DUCK HUNTERS. AS

YOU CAN DISCERN, ONE HUNTER IS VERY PROFICIENT, THE OTHER ONE DOES NOT SEEM TO BE VERY GOOD. EACH HUNTER HAS A DIFFERENT INDIVIDUAL VIEW OF THIS METHOD OF BAGGING DUCKS. I WOULD VENTURE TO SAY THAT THE GAME WARDEN WOULD PROBABLY HAVE IDEAS ABOUT THE HUNTER ON THE LEFT! IF THESE TWO GENTLEMEN WERE HUNTING DUCKS FOR A BOSS, THE BOSS WOULD HAVE STILL A THIRD VIEW OF THE DUCK HUNTER AS A WEAPONS SYSTEM. HE WOULD HAVE TO CONSIDER THE "GOOD" AND THE "BAD". WE MUST USE THE BOSS'S POINT OF VIEW IN OUR WAR GAMES, ALTHOUGH WE RECOGNIZE THAT THE INDIVIDUAL HUNTER'S POINT OF VIEW IS POSSIBLE, AND THAT SOME OF YOU MAY HAVE THIS POINT OF VIEW REGARDING SOME OF THE WEAPONS SYSTEMS IN THE FLEET. WE ASK THAT YOU BE MINDFUL THAT OTHER POINTS OF VIEW ALSO EXIST. THE INPUT DATA USED IN OUR WAR GAMES HERE ARE CAREFULLY CONSIDERED, AND WE BELIEVE THAT, IN MOST INSTANCES, THEY ARE REPRESENTATIVE AND VALID.

SLIDE 15 OFF

IV. THE BINOMIAL DISTRIBUTION.

NOW - LET US MOVE ON TO THE NEXT CONCEPT. IMAGINE THAT TWO SHOTS ARE FIRED AT A TARGET BY A FIXED GUN. ASSUME THAT THE PROBABILITY OF OBTAINING A HIT BY A SINGLE SHOT IS ONE-HALF, AND THAT THIS PROBABILITY REMAINS CONSTANT FROM SHOT TO SHOT AS THE FIRINGS ARE REPEATED.

NOW, IF TWO SUCH SHOTS ARE FIRED, ONE COULD OBSERVE THE NUMBER OF HITS OBTAINED. THE POSSIBILITIES ARE: ZERO HITS, ONE HIT, OR TWO HITS. THE QUESTION WE ASK NOW IS: "WHAT IS

THE PROBABILITY OF OBTAINING EXACTLY ZERO HITS, OR EXACTLY ONE HIT, OR EXACTLY TWO HITS?"

THERE IS A FORMULAE FOR COMPUTING THE ANSWERS TO THIS QUESTION.

SLIDE 16 ON

THE FORMULAE GIVES THE PROBABILITY THAT EXACTLY X HITS WILL BE OBTAINED, WHEN n SHOTS ARE FIRED, WHICH HAVE A SINGLE SHOT HIT PROBABILITY OF SMALL p. IN THE FORMULAE, SMALL p IS THE SINGLE SHOT HIT PROBABILITY, AND (1-p) IS THE PROBABILITY OF A MISS. n IS THE NUMBER OF SHOTS FIRED.

n, FOLLOWED BY AN EXCLAMATION MARK, MEANS n-FACTORIAL AND IS THE PRODUCT OF n, n-1, n-2, AND SO ON. FOR EXAMPLE, 3-FACTORIAL WOULD BE THE PRODUCT OF 3 TIMES 2 TIMES 1. ZERO-FACTORIAL IS 1 BY DEFINITION. x, OF COURSE, IS THE NUMBER OF HITS OBTAINED. THIS FORMULAE IS CALLED THE BINOMIAL DENSITY FUNCTION.

SLIDE 16 OFF

THE NEXT SLIDE WILL SHOW THE CALCULATIONS OF THE PROBABILITIES OF GETTING EXACTLY ZERO HITS, 1 HIT, AND 2 HITS, WHEN TWO SHOTS ARE FIRED FROM THE FIXED GUN.

SLIDE 17 ON

THE GRAPH, OR HISTOGRAM AS IT IS CALLED, IS THE BINOMIAL DENSITY GENERATED BY THE BINOMIAL DENSITY FUNCTION. THE CALCU-

LATIONS BELOW THE HISTOGRAM SHOW THAT THE PROBABILITY OF OBTAINING EXACTLY ZERO HITS IS .25, OF EXACTLY 1 HIT IS .50, AND OF EXACTLY 2 HITS IS .25. EVEN IF FIFTY SHOTS HAD BEEN FIRED, THERE STILL IS A PROBABILITY OF GETTING ZERO HITS. MANY WAR GAMERS OVERLOOK THIS FACT, AND WONDER WHY THEY GET NO HITS WHEN THEY FIRE, SAY, FIVE SHOTS.

SLIDE 17 OFF

NOTICE THAT I HAVE REPEATEDLY USED THE TERM "EXACTLY" 1 HIT, "EXACTLY" 2 HITS. IN ADDITION TO KNOWING THE PROBABILITY OF OBSERVING EXACTLY A SPECIFIED NUMBER OF HITS, IT IS SOMETIMES USEFUL TO KNOW THE PROBABILITIES OF OBSERVING "AT LEAST" OR "AT MOST", A CERTAIN NUMBER OF HITS. THESE PROBABILITIES ARE VERY EASY TO COMPUTE. CONSIDER AGAIN THE LAST EXAMPLE.

OBSERVING EXACTLY ONE HIT, FOR THE TWO SHOTS, IS CERTAINLY MUTUALLY EXCLUSIVE OF OBSERVING ZERO, OR 2 HITS. THESE EVENTS ARE MUTUALLY EXCLUSIVE.

SLIDE 18 ON

SINCE THE EVENTS ARE MUTUALLY EXCLUSIVE, THE PROBABILITY OF OBTAINING AT LEAST ONE HIT IS THE SUM OF THE PROBABILITIES OF OBTAINING EXACTLY ONE HIT AND OF OBTAINING EXACTLY TWO HITS. AT LEAST ONE HIT MEANS ONE, OR MORE, HITS. THE PROBABILITY OF OBTAINING AT MOST ONE HIT, WHICH IMPLIES ONE HIT, OR FEWER, IS THE SUM OF THE PROBABILITIES OF OBTAINING EXACTLY ONE HIT AND ZERO HITS. IN BOTH CASES, THE VALUE OF THE PROBABILITIES IS

THREE-FOURTHS. THESE PRINCIPLES CAN BE EXPANDED TO MORE SHOTS, AND MORE POSSIBLE OUTCOMES.

SLIDE 18 OFF

THIS CASE OF THE TWO SHOTS WAS A CASE OF THE BINOMIAL PROBABILITY DISTRIBUTION. THIS SPECIFIC CASE WAS THE BERNOULLI VERSION IN WHICH THE SINGLE SHOT HIT PROBABILITY REMAINS CONSTANT FROM TRIAL TO TRIAL. IF THE SINGLE SHOT HIT PROBABILITY VARIED FROM TRIAL TO TRIAL, SUBJECT TO CERTAIN CONSTRAINTS, THE POISSON PROBABILITY DISTRIBUTION WOULD RESULT. THERE ARE MANY OTHER PROBABILITY DISTRIBUTIONS, SUCH AS THE FREQUENTLY OCCURRING GAUSSIAN FUNCTION WHICH IS A CONTINUOUS PROBABILITY FUNCTION SOMETIMES USED TO APPROXIMATE THE BINOMIAL DISTRIBUTION. THE GAUSSIAN FUNCTION GENERATES THE FAMOUS "BELL-SHAPED" CURVE ABOUT WHICH YOU MAY HAVE HEARD. TIME DOES NOT PERMIT US TO LOOK AT ALL OF THESE DISTRIBUTIONS.

V. <u>EXPECTATION OF GAIN</u>.

ANOTHER CONCEPT USED IN GAMING ALSO INVOLVES THE USE OF PROBABILITY. IT IS THE EXPECTATION OF GAIN. SUPPOSE THAT AN EVENT MAY OCCUR WITH PROBABILITY P.

SLIDE 19 ON

IF THE EVENT OCCURS, THEN AN INDIVIDUAL WILL BE AWARDED A PRIZE IN THE AMOUNT W. THE EXPECTATION OF GAIN, G, IS P TIMES W. IF THERE ARE n EVENTS, THE EXPECTED GAIN IS GIVEN BY

THE SECOND RELATIONSHIP, PROVIDED THAT THE EVENTS ARE MUTUALLY EXCLUSIVE AND EXHAUSTIVE, THAT IS, ALL CASES ARE CONSIDERED. NOTE THAT THE EXPECTED GAIN FOR THESE MULTIPLE EVENTS IS SIMPLY THE SUM OF ALL INDIVIDUAL EXPECTED GAINS.

AS AN EXAMPLE OF THIS CONCEPT, RECONSIDER THE EXAMPLE OF THE PATROL SHIP IN THE MINED CHANNEL IN THE MUTUALLY EXCLUSIVE CASE. THAT CASE ASSUMED THAT IF THE SHIP HIT A MINE IT WOULD BE SUNK.

SLIDE 19 OFF, SLIDE 20 ON

ALSO RECALL THAT THE ABSOLUTE PROBABILITY THAT THE PATROL SHIP WILL HIT A SPECIFIED MINE WAS ONE-SIXTH. LOOKING AT THE PROBLEM FROM THE POINT OF VIEW OF THE PATROL SHIP COMMANDER, MINUS ONE WILL BE ASSIGNED IF THE SHIP IS MINED. THUS MINUS ONE WILL BE ASSIGNED FOR W. USING THE FORMULA FOR MUTUALLY EXCLUSIVE EVENTS, THE EXPECTED GAIN IS MINUS TWO-SIXTHS. THUS THE PATROL SHIP COMMANDER CAN EXPECT TO LOSE TWO OUT OF SIX PATROL SHIPS IN THIS CHANNEL. FROM A WAR GAMING POINT OF VIEW, IT IS SIMPLY NECESSARY TO DECIDE (WHICH TWO) OF ANY SIX PATROL VESSELS SENT THROUGH THE CHANNEL ARE SUNK. THIS DECISION COULD BE MADE BY DRAWING STRAWS TO DETERMINE (WHICH TWO) ARE THE ILL-FATED ONES.

SLIDE 20 OFF

VI. DEVICES USED FOR PROBABILISTIC DECISIONS IN WAR GAMING.

STRAWS ARE NOT COMONLY USED IN WAR GAMING, HOWEVER. LET US LOOK AT SOME OF THE CHANCE DEVICES THAT ARE USED IN WAR GAMING FOR MAKING PROBABILISTIC DECISIONS. THE MOST COMMON DEVICE IS AN ORDINARY PAIR OF DICE.

SLIDE 21 ON

THIS SLIDE SHOWS THE APPROXIMATE PROBABILITIES, IN STEPS OF ONE-TENTH, OF GETTING THE NUMBERS INDICATED ON THE DICE. WE CAN USE THIS TABLE IN THE FOLLOWING MANNER. A MISSILE HAS A PROBABILITY OF HITTING A TARGET OF THREE-TENTHS (UNDERLINED IN RED IN THE TABLE). WE WILL ROLL THE DICE TO SEE IF THE MISSILE HITS THE TARGET. IF THE NUMBERS ON THE DICE ADD UP TO A 6 OR A 7, WE AWARD THE HIT, OTHERWISE WE SAY THAT THE MISSILE MISSED. (PAUSE).

SLIDE 21 OFF

ICOSAHEDRON DICE, OR TWENTY-SIDED DICE ARE USED IN THE SAME WAY AS ORDINARY DICE, AND THEY CAN GIVE ACCURATE REPRESENTATIONS OF PROBABILITIES TO THREE DECIMAL PLACES. THEY HAVE FACES READING FROM 0 TO 9, REPEATED IDENTICALLY ON EACH DIE, AND CAN BE READ DIRECTLY WITHOUT REFERENCE TO A TABLE FOR INTERPRETATION. CONSIDERING THE MISSILE THAT HAD THE THREE-TENTHS HIT PROBABILITY, WE COULD HAVE ROLLED A TWENTY-SIDED DIE. IF THE DIE READ 1, 2, OR 3, WE WOULD HAVE AWARDED A HIT; OTHERWISE, A MISS.

OTHER EXAMPLES OF CHANCE DEVICES WHICH HAVE BEEN USED ARE:

SLIDE 22 ON

THE SPINNING POINTER; THIS ONE IS DIVIDED FOR A .5 PROBABILITY OF OCCURRENCE; ANOTHER IS A WELL-OILED ROLLER SKATE WHEEL WHERE NUMBERS ON THE PERIPHERY ARE READ ON AN INDEX; THE LAST IS A TABLE MADE UP FROM PREVIOUS DICE ROLLS. THIS ONE GIVES AN AFFIRMATIVE WITH A PROBABILITY OF .4.

SLIDE 22 OFF

THERE ARE OTHER DEVICES, YOU MAY BE SURE. PROBABLY THE MOST SOPHISTICATED DEVICE IS THE PROBABILITY GENERATING SECTION OF THE DAMAGE COMPUTER IN THE NEWS, WHICH IS A COMPLEX ELECTRONIC CIRCUIT.

IN CONCLUSION, LET US REVIEW SOME OF THE MORE IMPORTANT CONCEPTS WHICH YOU HAVE SEEN:

SLIDE 23 ON

PROBABILITY IS DEFINED THEORETICALLY AND EXPERIMENTALLY. IN WAR GAMING, EITHER DEFINITION IS <u>USUALLY</u> ACCEPTABLE.

MULTIPLE EVENTS CAN BE:

SIMULTANEOUS EVENTS, MUTUALLY EXCLUSIVE EVENTS, OR STATISTICALLY INDEPENDENT EVENTS.

WHEN OBSERVING MULTIPLE OCCURRENCES OF AN EVENT, ONE MAY OBSERVE <u>EXACTLY</u>, <u>AT LEAST</u>, OR <u>AT MOST</u>, A SPECIFIED NUMBER OF OCCURRENCES. THE PROBABILITIES OF THESE OBSERVATIONS ARE

DIFFERENT!

EXPECTED GAIN IS EQUAL TO THE PROBABILITY THAT AN EVENT OCCURS, TIMES THE WINNINGS IF THE EVENT DOES OCCUR, THAT IS, GAIN EQUALS P TIMES W.

SLIDE 23 OFF

LASTLY, BEAR IN MIND THAT ONE MUST PRECISELY NAME THE PROBABILITY TO WHICH HE IS REFERRING IN ORDER TO AVOID CONFUSION.